Physics 212 FINAL EXAMINATION 4 Feb. 2009

I - We work in the C.M. frame of a proton of mass $m = 940\ MeV/c^2$ and a deuteron of mass $2m$. The speed of the deuteron is $0.8\,c$

1 - What is the speed of the proton?

2 - What is the relative speed of the two particles?

II - 10^6 α-particles are scattered between $45°$ and $90°$. How many will be scattered between the same angles if their speed is doubled?

III - An electron placed in the potential

$$U(x) = 0 \quad \text{for} \quad 0 \le x \le L$$
$$= \infty \quad \text{elsewhere},$$

jumps from the state n_i to the state $n_f < n_i$. Show that the only allowed transitions occur for $\Delta n = n_f - n_i$ an odd number.

IV - A particle of mass m moves in the potential

$$U(x) = \frac{1}{2} m\omega^2 x^2$$

1 - Write down the Hamiltonian and the time-independent Schrödinger equation.

2 - The eigenfunction of the ground state has the form

$$\psi_0(x) = C\, e^{-\alpha x^2}$$

Determine the constants C, α and the energy E_0 of the ground state.

3 - If the wave function is $C\, e^{-\alpha(x-x_0)^2}$

a) Calculate the mean value $\overline{x}$. *

b) What is the probability that a measurement of energy yields $E = E_0$?

N.B. The identity $x^2 - x_0 x = \left(x - \frac{x_0}{2}\right)^2 - \frac{x_0^2}{4}$ is useful

Table of Integrals

$$\int_{-\infty}^{\infty} e^{-y^2}\, dy = \sqrt{\pi}$$

$$\int dy\, y \cos y = \cos y + y \sin y \,.$$